

# Multisensor Tracking of a Maneuvering Target in Clutter with Asynchronous Measurements using IMMPDA Filtering and Parallel Detection Fusion

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## Abstract

We present a (suboptimal) filtering algorithm for tracking a highly maneuvering target in a cluttered environment using multiple sensors dealing with possibly asynchronous (time delayed) measurements. The filtering algorithm is developed by applying the basic Interacting Multiple Model (IMM) approach, the Probabilistic Data Association (PDA) technique, and asynchronous measurement updating for state-augmented system estimation for the target. A state augmented approach is developed to estimate the time delay between local and remote sensors. A multi-sensor probabilistic data association filter is developed for parallel sensor processing for target tracking under clutter. The algorithm is illustrated via a highly maneuvering target tracking simulation example where two sensors, a radar and an infrared sensor, are used. Compared with an existing IMMPDA filtering algorithm with the assumption of synchronous (no delay) measurements sensor processing, the proposed algorithm achieves considerable improvement (especially in the case of larger delays) in the accuracy of track estimation.

**Keywords:** Asynchronous (Delayed) Measurements; Multisensor Parallel Updating; Interacting Multiple Model (IMM); Probabilistic Data Association (PDA).

## I Introduction

We consider the problem of tracking a single maneuvering target in clutter. This class of problem has received considerable attention in the literature [1, 2, 3, 4, 9]. In target tracking systems

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measurements are typically collected in "scans" or "frames" and then transmitted to a processing center [5, 6]. Asynchronous (delayed) measurements arise in a multisensor central tracking system due to communication network delays, varying preprocessing times at the sensor platforms and possibly lack of sampling time synchronization among sensor platforms. One of the asynchronous measurement problems is that of out-of-sequence measurements (OOSM) where measurements at various sensors may arrive out-of-sequence (not in correct time order) at the central processor. OOSM has been considered using interacting multiple model (IMM) [6, 7, 8]. In this paper we do not consider OOSM but, instead, consider "in-sequence" measurements with a fixed but unknown relative time-delay among sensor measurements. Various sensor measurements are assumed to be at the same rate but not necessarily time synchronized. All measurements over one sampling interval (based on the local clock of the central processor) are collected at the central processor, attributed to one time instant and processed simultaneously. We exploit interacting multiple model (IMM) and probabilistic data association (PDA) techniques. It is assumed that a track has been formed (initiated) and the objective of this work is to investigate fixed-but-unknown relative time-delay (measurement timing mismatch) arising in a multisensor central tracking system.

In [6], fixed-lag smoothing techniques have been investigated using IMM algorithm combined with PDA filter in a multiple sensor scenario to propose a combined IMMSPDAF (interacting multiple model multiple sensor probabilistic data association filter). We exploit the basic structure of [1] in combination with a state-augmented approach to deal with the fixed-but-unknown relative time-delay. In [1] and [14] it is assumed that the sensors are collocated and (time) synchronized with the sampling rate. In contrast, the sensor collocation and (time) synchronization are no longer assumed in this paper. Also, unlike [1, 9, 12] which have used sequential updating of the state estimates with measurements (i.e., updating of the state estimates sequentially with measurements from different sensors), we use parallel updating of the state estimates with measurements (i.e., updating of the state estimates with all measurements at the same time). For linear systems, the two updating methods are algebraically equivalent but for nonlinear filtering, the parallel updating can yield better performance in spite of higher computational cost [4]. Ref. [14] uses parallel updating but has some errors: during data association, all measurements at the same time from different sensors are assumed to be either from clutter or from the target. The possibility that a measurement from sensor 1 may be from target while the measurement from sensor 2 may be clutter-induced (and vice-versa) is implicitly not allowed in [14] – this is clearly incorrect. Ref. [10] allows for such distinctions (hypotheses), however, it is limited to non-maneuvering targets. In this paper, we also extend the multisensor approach of [10] to maneuvering targets (see Step 4 in Sec. IV).

The paper is organized as follows. Section II presents the problem formulation. Section III

describes the state-augmented system approach. Section IV describes the proposed IMMSPDAF algorithm for asynchronous measurements. Simulation results using the proposed algorithm for a realistic problem are given in Section V. Finally, Section VI presents a discussion of the results and some conclusions.

## II Problem Formulation

We assume that the target dynamics can be modeled by one of  $n$  hypothesized models. The model set is denoted as  $M^n := \{1, \dots, n\}$  and there are total  $q$  sensors. The event that model  $m$  is in effect during the sampling period  $(t_{k-1}, t_k]$  is denoted by  $M_k^m$ . For the  $m$ th hypothesized model (mode), the state dynamics and measurements, respectively, are modeled as

$$x_k = F_{k,k-1}^m x_{k-1} + G_{k,k-1}^m v_{k-1}^m \quad (1)$$

and

$$z_k^l = h^{m,l}(x_k) + w_k^{m,l} \quad \text{for } l = 1, \dots, q : \text{ local model at the sensor,} \quad (2)$$

where  $x_k$  is the system state at  $t_k$  and of dimension  $n_x$ ,  $z_k^l$  is the (true) measurement vector (i.e., due to the target) at sensor  $l$  at  $t_k$  and of dimension  $n_{z,l}$ ,  $F_{k,k-1}^m$  and  $G_{k,k-1}^m$  are the system matrices when model  $m$  is in effect over the sampling period  $(t_{k-1}, t_k]$ , and  $h^{m,l}$  is the nonlinear transformation of  $x_k$  to  $z_k^l$  ( $l = 1, \dots, q$ ) for model  $m$ . A first-order linearized version of (2) is given by

$$z_k^l = H_k^{m,l} x_k + w_k^{m,l} \quad \text{for } l = 1, \dots, q \quad (3)$$

where  $H_k^{m,l}$  is the Jacobian matrix of  $h^{m,l}$  evaluated at some value of the estimate of state  $x_k$  (see Sec. III). The process noise  $v_{k-1}^m$  and the measurement noise  $w_k^{m,l}$  are mutually uncorrelated zero-mean white Gaussian processes with covariance matrices  $Q_{k-1}^m$  and  $R_k^{m,l}$ , respectively. At the initial time  $t_0$ , the initial conditions for the system state under each model  $m$  are assumed to be Gaussian random variables with the known mean  $E\{x_0^m\}$  and the known covariance  $P_0^m$ . The probability of model  $m$  at  $t_0$ ,  $\mu_0^m = P\{M_0^m\}$ , is also known. The switching from model  $M_{k-1}^i$  to model  $M_k^m$  is governed by a finite-state stationary Markov chain with known transition probabilities  $p_{im} = P\{M_k^m | M_{k-1}^i\}$ . Henceforth, time  $t_k$  will be denoted by  $k$ .

Assume that there is a fixed but unknown relative time delay  $d_k$  (modulo  $T$ =sampling interval) at sample time  $t_k$  between the local sensor clock and the central processor clock at sample time  $t_k$ . [This time delay could be due to unsynchronized clocks at the two locations or due to inherent delay due to congestion, insufficient bandwidth etc. in the communication link between the remote sensor platform and the central processor.] The measurements from sensor  $l$  are sent to the central processor where all measurements collected between local sampling interval  $(t_{k-1}, t_k]$  are attributed

to time  $t_k$ . The state dynamics and measurements reported from the remote sensor platform at time  $t_{k_{dl}}$  (henceforth will be denoted by  $k_{dl}$ ) to the center processor at time  $t_k$  can be modeled as

$$x_{k_{dl}} = F_{k_{dl},k-1}^m x_{k-1} + G_{k_{dl},k-1}^m v_{k-1}^m \quad (4)$$

and

$$z_k^l = h^{m,l}(x_{k_{dl}}) + w_k^{m,l} \quad : \text{ model of the sensor at the central processor} \quad (5)$$

where  $t_{k_{dl}} = t_k - d_{kl}$  and  $d_{kl}$  is the time difference between the sampling time at the central processor and the measurement time at the local sensor (assume that  $0 \leq d_{kl} < T$ , where  $T$  is sampling time),  $x_{k_{dl}}$  is the system state at  $t_{k_{dl}}$  and of dimension  $n_x$ ,  $F_{k_{dl},k-1}^m$  and  $G_{k_{dl},k-1}^m$  are the system matrices when model  $m$  is in effect over the timing interval  $(t_{k-1}, t_{k_{dl}}]$

### III State-Augmented System

Define the augmented state  $\bar{x}_k$  from  $x_k$  as

$$\bar{x}_k = [x_k', v_k', x_{k-1}', v_{k-1}'] \quad (6)$$

where  $x_k'$  denotes the transpose of  $x_k$ . Assume that there is a fixed but unknown delay,  $d_{kl}$ , between the central processor and the remote sensor  $l$  platform. Using the above definitions (1, 6) and the measurement delay,  $d_{kl}$ , the augmented state equation may be written more compactly as

$$\bar{x}_k = \bar{F}_{k,k-1}^m \bar{x}_{k-1} + \bar{G}_{k,k-1}^m v_k^m \quad (7)$$

and

$$d_{kl} = d_{(k-1)l} + v_{k-1}^{dl} \quad (8)$$

where  $v_{k-1}^{dl}$  is a small processing noise assumed to be Gaussian noise with zero mean and (very) small but nonzero variance. Note that the process noise in (7) is  $v_k^m$  (at time  $k$  not at time  $k-1$ ).

Above equations (7) and (8) can also be absorbed into another augmented state  $\tilde{x}_k$  as

$$\tilde{x}_k = \begin{bmatrix} \bar{x}_k \\ d_{kl} \end{bmatrix} = \tilde{F}_{k,k-1}^m \tilde{x}_{k-1} + \tilde{G}_{k,k-1}^m \tilde{v}_k^m \quad \text{where} \quad \tilde{v}_k^m = \begin{bmatrix} v_k^m \\ v_{k-1}^{dl} \end{bmatrix}, \quad (9)$$

$\tilde{x}_k' = [x_k', v_k', x_{k-1}', v_{k-1}', d_{kl}]$ , and  $\tilde{F}_{k,k-1}$  and  $\tilde{G}_{k,k-1}$  are defined in Sec. V (see (46)-(53)). Using the augmented state (9) the counterparts to (2) and (5), respectively, are

$$z_k^l = h^{m,l}(\tilde{x}_k) + w_k^{m,l} = h^{m,l}([I, 0, 0, 0, 0] \tilde{x}_k) + w_k^{m,l} \quad (10)$$

and

$$z_k^l = h^{m,l}(\tilde{x}_{k_l}) + w_k^{m,l} = h^{m,l}([0, 0, F_{k_{dl},k-1}^m, G_{k_{dl},k-1}^m, 0] \tilde{x}_k) + w_k^{m,l} \quad (11)$$

for both measurements from local sensor and from remote sensor, respectively. To keep the notations and details to a bare minimum, *we will consider the case of two sensors only* and furthermore, we will assume that one of the sensors is either collocated with or is synchronized with the central processor, so that we will drop the subscript  $l$  from  $d_{kl}$ . For more than two sensors, we need to augment  $\tilde{x}_k$  with additional  $d_k$ 's (total  $q - 1$ ): in essence, these delays are relative to one of the sensors (reference sensor).

The following notations and definitions are used regarding the measurements at sensor  $l$ . Note that, in general, at any time some measurements may be due to clutter and some due to the target, i.e. there can be more than a single measurement at time  $k$  at sensor  $l$ . The measurement set (not yet validated) generated by sensor  $l$  at time  $k$  is denoted as

$$Z_k^l := \{z_k^{l(1)}, z_k^{l(2)}, \dots, z_k^{l(m_l)}\} \quad (12)$$

where  $m_l$  is the number of measurements generated by sensor  $l$  at time  $k$ . Variable  $z_k^{l(i)}$  ( $i = 1, \dots, m_l$ ) is the  $i$ th measurement within this set. The validated set of measurements of sensor  $l$  at time  $k$  will be denoted by  $Y_k^l$ , containing  $\bar{m}_l (\leq m_l)$  measurement vectors. The cumulative set of validated measurements from sensor  $l$  up to time  $k$  is denoted as

$$Z^{k(l)} := \{Y_1^l, Y_2^l, \dots, Y_k^l\}. \quad (13)$$

The cumulative set of validated measurements from all sensors up to time  $k$  is denoted as

$$Z^k := \{Z^{k(1)}, Z^{k(2)}, \dots, Z^{k(q)}\} \quad (14)$$

where  $q$  is the number of sensors.

Our goal is to find the state estimate

$$\hat{\tilde{x}}_{k|k} := E\{\tilde{x}_k | Z^k\} \quad (15)$$

and the associated error covariance matrix

$$\tilde{P}_{k|k} := E\{[\tilde{x}_k - \hat{\tilde{x}}_{k|k}][\tilde{x}_k - \hat{\tilde{x}}_{k|k}]' | Z^k\} \quad (16)$$

where  $x'_k$  denotes the transpose of  $x_k$ .

## IV IMMSPDAF Algorithm for Asynchronous Measurements

We now modify the IMM/(J)PDA algorithms of [9] and [12] to apply to the multi-sensor asynchronous measurements system. We confine our attention to the case of 2 sensors; however, the algorithm can be easily adapted to the case of arbitrary  $q$  sensors. We will only briefly outline the basic steps in "one cycle" (i.e., processing needed to update for a new set of measurements) of the IMMSPDA filter.

*Assumed available:* Given the state estimate  $\hat{x}_{k-1|k-1}^m := E\{\tilde{x}_{k-1}|M_{k-1}^m, Z^{k-1}\}$ , the associated covariance  $\tilde{P}_{k-1|k-1}^m$ , and the conditional mode probability  $\mu_{k-1}^m := P[M_{k-1}^m|Z^{k-1}]$  at time  $k-1$  for each mode  $m \in M^n$ .

**Step 1. Interaction – mixing of the estimate from the previous time** ( $\forall m \in M^n$ ) :

predicted mode probability:

$$\mu_k^{m-} := P[M_k^m|Z^{k-1}] = \sum_i p_{im} \mu_{k-1}^i. \quad (17)$$

mixing probability:

$$\mu^{i|m} := P[M_{k-1}^i|M_k^m, Z^{k-1}] = p_{im} \mu_{k-1}^i / \mu_k^{m-}. \quad (18)$$

mixed estimate:

$$\hat{x}_{k-1|k-1}^{0m} := E\{\tilde{x}_{k-1}|M_k^m, Z^{k-1}\} = \sum_i \hat{x}_{k-1|k-1}^i \mu^{i|m}. \quad (19)$$

covariance of the mixed estimate:

$$\begin{aligned} \tilde{P}_{k-1|k-1}^{0m} &:= E\{[\tilde{x}_{k-1} - \hat{x}_{k-1|k-1}^{0m}][\tilde{x}_{k-1} - \hat{x}_{k-1|k-1}^{0m}]'|M_k^m, Z^{k-1}\} \\ &= \sum_i \{\tilde{P}_{k-1|k-1}^i + [\hat{x}_{k-1|k-1}^i - \hat{x}_{k-1|k-1}^{0m}][\hat{x}_{k-1|k-1}^i - \hat{x}_{k-1|k-1}^{0m}]'\} \mu^{i|m}. \end{aligned} \quad (20)$$

**Step 2. Predicted state and measurements for sensors 1 and 2** ( $\forall m \in M^n$ ) :

state prediction:

$$\hat{x}_{k|k-1}^m := E\{\tilde{x}_k|M_k^m, Z^{k-1}\} = \tilde{F}_{k-1}^m \hat{x}_{k-1|k-1}^{0m}. \quad (21)$$

state prediction error covariance:

$$\tilde{P}_{k|k-1}^m := E\{[\tilde{x}_k - \hat{x}_{k|k-1}^m][\tilde{x}_k - \hat{x}_{k|k-1}^m]'|M_k^m, Z^{k-1}\} = \tilde{F}_{k-1}^m \tilde{P}_{k-1|k-1}^{0m} \tilde{F}_{k-1}^{m'} + \tilde{G}_{k-1}^m Q_{k-1}^m \tilde{G}_{k-1}^{m'}. \quad (22)$$

The mode-conditioned predicted measurement for sensor  $l$  is

$$\hat{z}_k^{m,l} := h^{m,l}(\hat{x}_{k|k-1}^m). \quad (23)$$

Using the linearized version (3), the covariance of the mode-conditioned residual

$$\nu_k^{m,l(i)} := z_k^{l(i)} - \hat{z}_k^{m,l},$$

is given by (assume  $q=2$ , the case of 2 sensors)

$$S_k^{m,1} := E\{\nu_k^{m,1(i)} \nu_k^{m,1(i)'} | M_k^m, Z^{k-1}\} = \tilde{H}_k^{m,1} \tilde{P}_{k|k-1}^m \tilde{H}_k^{m,1'} + R_k^{m,1}, \quad (24)$$

$$S_k^{m,2} := E\{\nu_k^{m,2(i)} \nu_k^{m,2(i)'} | M_k^m, Z^{k-1}\} = \tilde{H}_k^{m,2} \tilde{P}_{k|k-1}^m \tilde{H}_k^{m,2'} + R_k^{m,2} \quad (25)$$

where  $\tilde{H}_k^{m,l}$  is the first order derivative (Jacobian matrix) of  $h^{m,l}(\cdot)$  evaluated at the state prediction  $\hat{x}_{k|k-1}^m$  (see (23)). Note that (24) and (25) assume that  $z_k^{l(i)}$  originates from the target. The results (24) and (25) do not depend upon the actual measurements.

As mentioned earlier, since our approach to the problem deals not only with the asynchronous measurements but also with multiple simultaneous measurements [10, 11] arising from two separate sensors that are tracking a single target through a common surveillance region, a method for fusion of multiple measurements has to be devised. In order to do this, now the combined covariance  $S_k^m$  of the mode-conditioned residual obtained from (24) and (25) also needs to be considered as follows

$$S_k^m := \begin{bmatrix} \tilde{H}_k^{m,1} \\ \tilde{H}_k^{m,2} \end{bmatrix} \tilde{P}_{k|k-1}^m \begin{bmatrix} \tilde{H}_k^{m,1'} & \tilde{H}_k^{m,2'} \end{bmatrix} + \begin{bmatrix} R_k^{m,1} & 0 \\ 0 & R_k^{m,2} \end{bmatrix}. \quad (26)$$

**Step 3. Measurement validation for sensors 1 and 2 ( $\forall m \in M^n$ ) :**

There is uncertainty regarding the measurements' origins. Therefore, we perform validation for each target separately. One sets up a validation gate for sensor  $l$  centered at the mode-conditioned predicted measurement,  $\hat{z}_k^{m,l}$ . Let  $(|A| = \det(A))$

$$m_a := \arg \left\{ \max_{m \in M^n} |S_k^{m,l}| \right\}.$$

Then measurement  $z_k^{l(i)}$  ( $i=1,2,\dots,m_l$ ) is validated if and only if

$$[z_k^{l(i)} - \hat{z}_k^{m_a,l}]' [S_k^{m_a,l}]^{-1} [z_k^{l(i)} - \hat{z}_k^{m_a,l}] < \gamma \quad (27)$$

where  $\gamma$  is an appropriate threshold. The volume of the validation region with the threshold  $\gamma$  is

$$V_k^l := c_{n_{zl}} \gamma^{n_{zl}/2} |S_k^{m_a,l}|^{1/2} \quad (28)$$

where  $n_{zl}$  is the dimension of the measurement and  $c_{n_{zl}}$  is the volume of the unit hypersphere of this dimension ( $c_1 = 2$ ,  $c_2 = \pi$ ,  $c_3 = 4\pi/3$ , etc.). Choice of  $\gamma$  is discussed in more detail in [4, Sec. 2.3.2]. After performing the validation for each target separately, we deal with all the validated data for measurement fusion.

**Step 4. State estimation with validated measurement from sensors 1 and 2 ( $\forall m \in M^n$ ) :**

From among all the raw measurements from sensor  $l$  at time  $k$ , i.e.  $Z_k^l := \{z_k^{l(1)}, z_k^{l(2)}, \dots, z_k^{l(m_l)}\}$ , define the set of validated measurement for sensor  $l$  at time  $k$  as

$$Y_k^l := \{y_k^{l(1)}, y_k^{l(2)}, \dots, y_k^{l(\bar{m}_l)}\} \quad (29)$$

where  $\bar{m}_l$  is total number of validated measurement for sensor  $l$  at time  $k$  and

$$y_k^{l(i)} := z_k^{l(i)} \quad (30)$$

where  $1 \leq l_1 < l_2 < \dots < l_{\bar{m}_l} \leq m_l$  when  $\bar{m}_l \neq 0$ . Define the association events (hypotheses)  $\theta_k^{i,j}$  as follows (here we follow [10])



- $\theta_k^{0,0}$ : none of the measurements in  $Y_k^1$  or  $Y_k^2$  is target originated.
- $\theta_k^{0,j}$ : only  $y_k^{2(j)}$  in  $Y_k^2$  is a target measurement, all other measurements in  $Y_k^1$  or  $Y_k^2$  are clutter,  $i = 0, j = 1, \dots, \bar{m}_2$ .
- $\theta_k^{i,0}$ : only  $y_k^{1(i)}$  in  $Y_k^1$  is a target measurement, all other measurements in  $Y_k^1$  or  $Y_k^2$  are clutter,  $i = 1, \dots, \bar{m}_1, j = 0$ .
- $\theta_k^{i,j}$ :  $y_k^{1(i)}$  and  $y_k^{2(j)}$  in  $Y_k^1$  and  $Y_k^2$ , respectively, are target measurements, all other measurements are clutter,  $i = 1, \dots, \bar{m}_1, j = 1, \dots, \bar{m}_2$ .

Therefore, there are a total of  $\bar{m}_1\bar{m}_2 + \bar{m}_1 + \bar{m}_2 + 1$  possible association hypotheses, each of which has an association probability. Define the mode-conditioned association event probabilities as

$$\beta_k^{m,i,j} := P\{\theta_k^{i,j} | M_k^m, Y_k^1, Y_k^2, Z^{k-1}\}. \quad (31)$$

Exploiting the diffuse model for clutter in [1, 4], it turns out that

$$\begin{aligned} \beta_k^{m,0,0} &= C \frac{(1-P_{D_1}P_{G_1})(1-P_{D_2}P_{G_2})}{(V_k^1)^{\bar{m}_1}(V_k^2)^{\bar{m}_2}}, \quad i = 0, j = 0 \\ \beta_k^{m,0,j} &= C \frac{P_{D_2}(1-P_{D_1}P_{G_1})N[\nu_k^{m,2(j)}; 0, S_k^{m,2}]}{(V_k^2)^{\bar{m}_2-1}\bar{m}_2}, \quad i = 0, j = 1, \dots, \bar{m}_2 \\ \beta_k^{m,i,0} &= C \frac{P_{D_1}(1-P_{D_2}P_{G_2})N[\nu_k^{m,1(i)}; 0, S_k^{m,1}]}{(V_k^1)^{\bar{m}_1-1}\bar{m}_1}, \quad i = 1, \dots, \bar{m}_1, j = 0 \\ \beta_k^{m,i,j} &= C \frac{N[\nu_k^{m,1(i)}; 0, S_k^{m,1}]N[\nu_k^{m,2(j)}; 0, S_k^{m,2}]}{\bar{m}_1\bar{m}_2(V_k^1)^{\bar{m}_1-1}(V_k^2)^{\bar{m}_2-1}} P_{D_1}P_{D_2}, \quad i = 1, \dots, \bar{m}_1, j = 1, \dots, \bar{m}_2 \end{aligned} \quad (32)$$

where  $P_{D_1}$  and  $P_{D_2}$  are the detection probabilities that the sensors 1 and 2 detect the target, respectively,  $P_{G_1}$  and  $P_{G_2}$  are probabilities the target is in the validation region observed from sensors 1 and 2, respectively,  $C$  is a normalization constant such that  $\sum_{i=0}^{\bar{m}_1} \sum_{j=0}^{\bar{m}_2} \beta_k^{m,i,j} = 1 \forall m$  and

$$N[x; y, P] := |2\pi P|^{-1/2} \exp \left[ -\frac{1}{2} (x - y)' P^{-1} (x - y) \right].$$



Define the mode-conditioned innovations  $\nu_k^{m,i,j}$  as

$$\begin{aligned}
\nu_k^{m,0,0} &= \begin{bmatrix} 0_{n_{z1} \times 1} \\ 0_{n_{z2} \times 1} \end{bmatrix}, \quad i=0, j=0 \\
\nu_k^{m,0,j} &= \begin{bmatrix} 0_{n_{z1} \times 1} \\ \nu_k^{m,2(j)} \end{bmatrix}, \quad i=0, j=1, \dots, \bar{m}_2 \\
\nu_k^{m,i,0} &= \begin{bmatrix} \nu_k^{m,1(i)} \\ 0_{n_{z2} \times 1} \end{bmatrix}, \quad i=1, \dots, \bar{m}_1, j=0 \\
\nu_k^{m,i,j} &= \begin{bmatrix} \nu_k^{m,1(i)} \\ \nu_k^{m,2(j)} \end{bmatrix}, \quad i=1, \dots, \bar{m}_1, j=1, \dots, \bar{m}_2.
\end{aligned} \tag{33}$$

The likelihood function for each mode  $m$  is

$$\Lambda_k^m := p[Y_k^1, Y_k^2 | M_k^m, Z^{k-1}] = \sum_{i=0}^{\bar{m}_1} \sum_{j=0}^{\bar{m}_2} p[Y_k^1, Y_k^2, \theta_k^{i,j} | M_k^m, Z^{k-1}] \tag{34}$$

where

$$\begin{aligned}
p[Y_k^1, Y_k^2, \theta_k^{i,j} | M_k^m, Z^{k-1}] &= p[Y_k^1, Y_k^2 | M_k^m, \theta_k^{i,j}, Z^{k-1}] P[\theta_k^{i,j}] \\
&= \begin{cases} \frac{(1-P_{D_1}P_{G_1})(1-P_{D_2}P_{G_2})}{[V_k^1]^{\bar{m}_1}[V_k^2]^{\bar{m}_2}}, & i=0, j=0 \\ \frac{(1-P_{D_1}P_{G_1})(P_{D_2}P_{G_2})/\bar{m}_2}{P_{G_2}[V_k^2]^{\bar{m}_2-1}} \times N[\nu_k^{m,2(j)}; 0, S_k^{m,2}], & i=0, j=1, \dots, \bar{m}_2 \\ \frac{(P_{D_1}P_{G_1})(1-P_{D_2}P_{G_2})/\bar{m}_1}{P_{G_1}[V_k^1]^{\bar{m}_1-1}} \times N[\nu_k^{m,1(i)}; 0, S_k^{m,1}], & i=1, \dots, \bar{m}_1, j=0 \\ \frac{(P_{D_1}P_{G_1})(P_{D_2}P_{G_2})/(\bar{m}_1\bar{m}_2)}{P_{G_1}[V_k^1]^{\bar{m}_1-1}P_{G_2}[V_k^2]^{\bar{m}_2-1}} \times N[\nu_k^{m,i,j}; 0, S_k^m], & i=1, \dots, \bar{m}_1, j=1, \dots, \bar{m}_2. \end{cases} \tag{35}
\end{aligned}$$

Using  $\hat{x}_{k|k-1}^m$  (from (21)) and its covariance  $\tilde{P}_{k|k-1}^m$  (from (22)), one computes the partial update  $\hat{x}_{k|k}^m$  and its covariance  $\tilde{P}_{k|k}^m$  according to the standard PDAF [1], except that the augmented state is conditioned on  $\theta_k^{i,j}$  with data fusion from sensors 1 and 2. Define the combined mode-conditioned innovations

$$\nu_k^m := \sum_{i=0}^{\bar{m}_1} \sum_{j=0}^{\bar{m}_2} \beta_k^{m,i,j} \nu_k^{m,i,j}. \tag{36}$$

Therefore, partial update of the state estimate

$$\hat{x}_{k|k}^{m,i,j} := E\{x_k | \theta_k^{i,j}, M_k^m, Z^{k-1}, Y_k^1, Y_k^2\} = \hat{x}_{k|k-1}^m + W_k^{m,i,j} \nu_k^{m,i,j} \tag{37}$$

where Kalman gains,  $W_k^{m,i,j}$ , are computed as

$$= \begin{cases} W_k^{m,0,0} = 0, & \text{for } i = 0, j = 0 \\ W_k^{m,i,0} = \tilde{P}_{k|k-1}^m [\tilde{H}_k^{m,1'} [S_k^{m,1}]^{-1} \ 0], & \text{for } i \neq 0, j = 0 \\ W_k^{m,0,j} = \tilde{P}_{k|k-1}^m [0 \ \tilde{H}_k^{m,2'} [S_k^{m,2}]^{-1}], & \text{for } i = 0, j \neq 0 \\ W_k^{m,i,j} = \tilde{P}_{k|k-1}^m \tilde{H}_k^{m'} [S_k^m]^{-1}, & \text{for } i \neq 0, j \neq 0, \end{cases} \quad (38)$$

and  $\tilde{H}_k^{m'} = [\tilde{H}_k^{m,1'} \ \tilde{H}_k^{m,2'}]$ . Therefore, mode-conditioned update of the state estimate

$$\hat{x}_{k|k}^m := E \{x_k | M_k^m, Z^{k-1}, Y_k^1, Y_k^1\} = \sum_{i=0}^{\tilde{m}_1} \sum_{j=0}^{\tilde{m}_2} \beta_k^{m,i,j} \hat{x}_{k|k-1}^{m,i,j} \quad (39)$$

and covariance of  $\hat{x}_{k|k}^m$

$$\begin{aligned} \tilde{P}_{k|k}^m &:= \tilde{P}_{k|k-1}^m - \sum_{i=0, (i,j) \neq (0,0)}^{\tilde{m}_1} \sum_{j=0}^{\tilde{m}_2} \beta_k^{m,i,j} W_k^{m,i,j} S_k^{m,i,j} W_k^{m,i,j'} \\ &+ \sum_{i=0}^{\tilde{m}_1} \sum_{j=0}^{\tilde{m}_2} \beta_k^{m,i,j} W_k^{m,i,j} \nu_k^{m,i,j} \nu_k^{m,i,j'} W_k^{m,i,j'} \\ &- \left[ \sum_{i=0}^{\tilde{m}_1} \sum_{j=0}^{\tilde{m}_2} \beta_k^{m,i,j} W_k^{m,i,j} \nu_k^{m,i,j} \right] \left[ \sum_{i=0}^{\tilde{m}_1} \sum_{j=0}^{\tilde{m}_2} \beta_k^{m,i,j} W_k^{m,i,j} \nu_k^{m,i,j} \right]'. \end{aligned} \quad (40)$$

**Step 5. Update of mode probabilities** ( $\forall m \in M^n$ ) :

$$\mu_k^m := P[M_k^m | Z^k] = \frac{1}{C} \mu_k^{m-} \Lambda_k^m \quad (41)$$

where  $C$  is a normalization constant such that  $\sum_m \mu_k^m = 1$ .

**Step 6 Combination of the mode-conditioned estimates** ( $\forall m \in M^n$ ) : The final augmented state estimate update at time  $k$  is given by

$$\hat{x}_{k|k} = \sum_m \hat{x}_{k|k}^m \mu_k^m \quad (42)$$

and its covariance is given by

$$\tilde{P}_{k|k} = \sum_m \left\{ \tilde{P}_{k|k}^m + [\hat{x}_{k|k}^m - \hat{x}_{k|k}] [\hat{x}_{k|k}^m - \hat{x}_{k|k}]' \right\} \mu_k^m. \quad (43)$$

From the final augmented state (see (42)), the state filtered vector  $\hat{x}_{k|k}$  and the state smoothing vector  $\hat{x}_{k-1|k}$  can be easily obtained.

## V Simulation Example

The following example of tracking a highly maneuvering target in clutter is considered. The target starts at location [21689 10840 40] in Cartesian coordinates in meters. The initial velocity (in m/s) is [-8.3 -399.9 0] and the target stays at constant altitude with a constant speed of 400 m/s. Its

trajectory is a straight line with constant velocity between 0 and 20s, a coordinated turn (0.15 rad/s) with constant acceleration of 60 m/s<sup>2</sup> between 20 and 35s, a straight line with constant velocity between 35 and 55s, a coordinated turn (0.1 rad/s) with constant acceleration of 40 m/s<sup>2</sup> between 55 and 70s, and a straight line with constant velocity between 70 and 90s. The target motion models are patterned and modified after [1]. In each mode the target dynamics are modeled in Cartesian coordinates as

$$\tilde{x}_k = \tilde{F}_{k,k-1}^m \tilde{x}_{k-1} + \tilde{G}_{k,k-1}^m \tilde{v}_k^m \quad (44)$$

$$\tilde{x}_{k_d} = \tilde{F}_{k_d,k-1}^m \tilde{x}_{k-1} + \tilde{G}_{k_d,k-1}^m \tilde{v}_k^m \quad (45)$$

where the augmented state of the target consists of position, velocity, acceleration, and the process noise in each of the three Cartesian coordinates ( $x, y$ , and  $z$ ) at  $t_k$  and  $t_{k-1}$  as well as the delay time  $d_k$  at  $t_k$ . Thus both  $\tilde{x}_k$  and  $\tilde{x}_{k_d}$  are of dimension 25 ( $n_x = 25$ ). Three maneuver models are considered in the following discussion. The system matrices  $\tilde{F}_{k,k-1}^m$ ,  $\tilde{G}_{k,k-1}^m$ ,  $\tilde{F}_{k_d,k-1}^m$  and  $\tilde{G}_{k_d,k-1}^m$  are defined as

$$\tilde{F}_{k,k-1}^m = \begin{bmatrix} \bar{F}_{k,k-1}^m & 0 \\ 0 & I \end{bmatrix}, \quad \tilde{G}_{k,k-1}^m = \begin{bmatrix} \bar{G}_{k,k-1}^m & 0 \\ 0 & I \end{bmatrix} \quad (46)$$

$$\tilde{F}_{k_d,k-1}^m = \begin{bmatrix} \bar{F}_{k_d,k-1}^m & 0 \\ 0 & I \end{bmatrix}, \quad \tilde{G}_{k_d,k-1}^m = \begin{bmatrix} \bar{G}_{k_d,k-1}^m & 0 \\ 0 & I \end{bmatrix} \quad (47)$$

where

$$\bar{F}_{k,k-1}^m = \begin{bmatrix} F_{k,k-1}^m & G_{k,k-1}^m \\ 0 & 0 \end{bmatrix}, \quad \bar{G}_{k,k-1}^m = \begin{bmatrix} 0 \\ I \end{bmatrix} \quad (48)$$

$$\bar{F}_{k_d,k-1}^m = \begin{bmatrix} F_{k_d,k-1}^m & G_{k_d,k-1}^m \\ 0 & 0 \end{bmatrix}, \quad \bar{G}_{k_d,k-1}^m = \begin{bmatrix} 0 \\ I \end{bmatrix}, \quad (49)$$

$$F_{k,k-1}^m = \begin{bmatrix} F^m & 0 & 0 \\ 0 & F^m & 0 \\ 0 & 0 & F^m \end{bmatrix}, \quad G_{k,k-1}^m = \begin{bmatrix} G^m & 0 & 0 \\ 0 & G^m & 0 \\ 0 & 0 & G^m \end{bmatrix}, \quad (50)$$

$$F_{k_d,k-1}^m = \begin{bmatrix} F_d^m & 0 & 0 \\ 0 & F_d^m & 0 \\ 0 & 0 & F_d^m \end{bmatrix}, \quad G_{k_d,k-1}^m = \begin{bmatrix} G_d^m & 0 & 0 \\ 0 & G_d^m & 0 \\ 0 & 0 & G_d^m \end{bmatrix}. \quad (51)$$

**Model 1.** Nearly constant velocity model with zero mean perturbation in acceleration

$$F^1 = \begin{bmatrix} 1 & T & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad G^1 = \begin{bmatrix} \frac{T^2}{2} \\ T \\ 0 \end{bmatrix}, \quad (52)$$

$$F_d^1 = \begin{bmatrix} 1 & (T - d_k) & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad G_d^1 = \begin{bmatrix} \frac{(T - d_k)^2}{2} \\ (T - d_k) \\ 0 \end{bmatrix}, \quad (53)$$

where  $T$  is the sampling period. The standard deviation of the process noise of  $M^1$  is 5 m/s<sup>2</sup> (as in [1]).

**Model 2.** Wiener process acceleration (nearly constant acceleration motion)

$$F^2 = \begin{bmatrix} 1 & T & \frac{T^2}{2} \\ 0 & 1 & T \\ 0 & 0 & 1 \end{bmatrix}, \quad G^2 = \begin{bmatrix} \frac{T^2}{2} \\ T \\ 1 \end{bmatrix}, \quad (54)$$

$$F_d^2 = \begin{bmatrix} 1 & (T - d_k) & \frac{(T - d_k)^2}{2} \\ 0 & 1 & (T - d_k) \\ 0 & 0 & 1 \end{bmatrix}, \quad G_d^2 = \begin{bmatrix} \frac{(T - d_k)^2}{2} \\ (T - d_k) \\ 1 \end{bmatrix}. \quad (55)$$

The standard deviation of the process noise of  $M^2$  is 7.5 m/s<sup>2</sup> (as in [1]).

**Model 3.** Wiener process acceleration (model with large acceleration increments, for the onset and termination of maneuvers), with  $F^3 = F^2$ ,  $G^3 = G^2$ ,  $F_d^3 = F_d^2$  and  $G_d^3 = G_d^2$ . The standard deviation of the process noise of  $M^3$  is 40 m/s<sup>2</sup> (as in [1]).

The initial model probabilities are  $\mu_0^1 = 0.8$ ,  $\mu_0^2 = 0.1$  and  $\mu_0^3 = 0.1$ . The mode switching probability matrix is given by (as in [1])

$$\begin{bmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{bmatrix} = \begin{bmatrix} 0.8 & 0.0 & 0.2 \\ 0.0 & 0.8 & 0.2 \\ 0.3 & 0.3 & 0.4 \end{bmatrix}. \quad (56)$$

**The Sensors:** Two sensors are used to obtain the measurements. Sensor 1 and Sensor 2 are located at  $[x_1, y_1, z_1] = [-4000 \ 4000 \ 0]$  m and  $[x_2, y_2, z_2] = [5000 \ 0 \ 0]$  m, respectively, and the central processor is collocated with sensor 1 platform (we assume that there is no time delay between sensor

1 and central processor and there is fixed but unknown time delay between sensor 2 and central processor). The measurements from sensor  $l$  for model  $m$  are  $z_k^l = h^{m,l}(x_k) + w_k^{m,l}$  for  $l = 1$  and 2, reflecting range and azimuth angle for sensor 1 (radar) and azimuth and elevation angles for sensor 2 (infrared). The range, azimuth, and elevation angle transformations, respectively, are given by

$$r_l = \{(x - x_l)^2 + (y - y_l)^2 + (z - z_l)^2\}^{1/2} \quad (57)$$

$$a_l = \tan^{-1}[(y - y_l)/(x - x_l)] \quad (58)$$

$$e_l = \tan^{-1}[(z - z_l)/\{(x - x_l)^2 + (y - y_l)^2\}^{1/2}]. \quad (59)$$

As we see from (1), (2), (4) and (5), the measurements obtained from sensors 1 and 2 can be expressed as

$$z_k^1 = h^1([I, 0, 0, 0, 0]\tilde{x}_k) + w_k^1 \quad (60)$$

$$z_k^2 = h^2([0, 0, F_{k_d, k-1}^m, G_{k_d, k-1}^m, 0]\tilde{x}_k) + w_k^2. \quad (61)$$

The measurement noise  $w_k^{m,l}$  for sensor  $l$  is assumed to be zero-mean white Gaussian with known covariances,  $R^1 = \text{diag}[q_r, q_{a1}] = \text{diag}[400m^2, 49mrad^2]$  with  $q_r$  and  $q_{a1}$  denoting the variances for the radar range and azimuth measurement noises, respectively, and  $R^2 = \text{diag}[q_{a2}, q_e] = \text{diag}[4mrad^2, 4mrad^2]$  with  $q_{a2}$  and  $q_e$  denoting the variances for the infrared sensor azimuth and elevation measurement noises, respectively. The sampling interval was  $T=1s$  and it was assumed that the probability of detection  $P_d=1$  for both sensors.

**The Clutter:** For generating false measurements in simulations, the clutter was assumed to be Poisson distributed with expected number of  $\lambda_1 = 13 \times 10^{-6}/m \text{ mrad}$  for sensor 1 and  $\lambda_2 = 7 \times 10^{-4}/m \text{ mrad}$  for sensor 2 [1, case 1]. These statistics were used for generating the clutter in all simulations. However, a nonparametric clutter model was used for implementing all the algorithms for target tracking.

**Other Parameters:** The gates for setting up the validation regions for both the sensors were based on the threshold  $\gamma=16$ . With the measurement vector of dimension 2, this leads to a gate probability  $P_G=0.997$  (see [4, pages 95-96]).

**Simulation Results:** The results were obtained from 100 Monte Carlo runs. Fig. 1 shows the true trajectory of the target. Fig. 2 shows the delay estimates (given unknown but fixed timing mismatch between the two sensors) based on 100 Monte Carlo runs. Fig. 3 shows the RMSE (root mean-square error) for the filtered state and the smoothed state (lag =1) in position, velocity and acceleration. It is seen from Fig. 3 that the smoothing method shows better accuracy than the filtering method as well described in [9]. Fig. 4 shows a comparison among the performances of the proposed IMMSPDAF algorithm dealing with asynchronous measurements with unknown but fixed  $d_k$ , with known  $d$ , and the standard IMMSPDAF algorithm with the assumption that  $d=0$

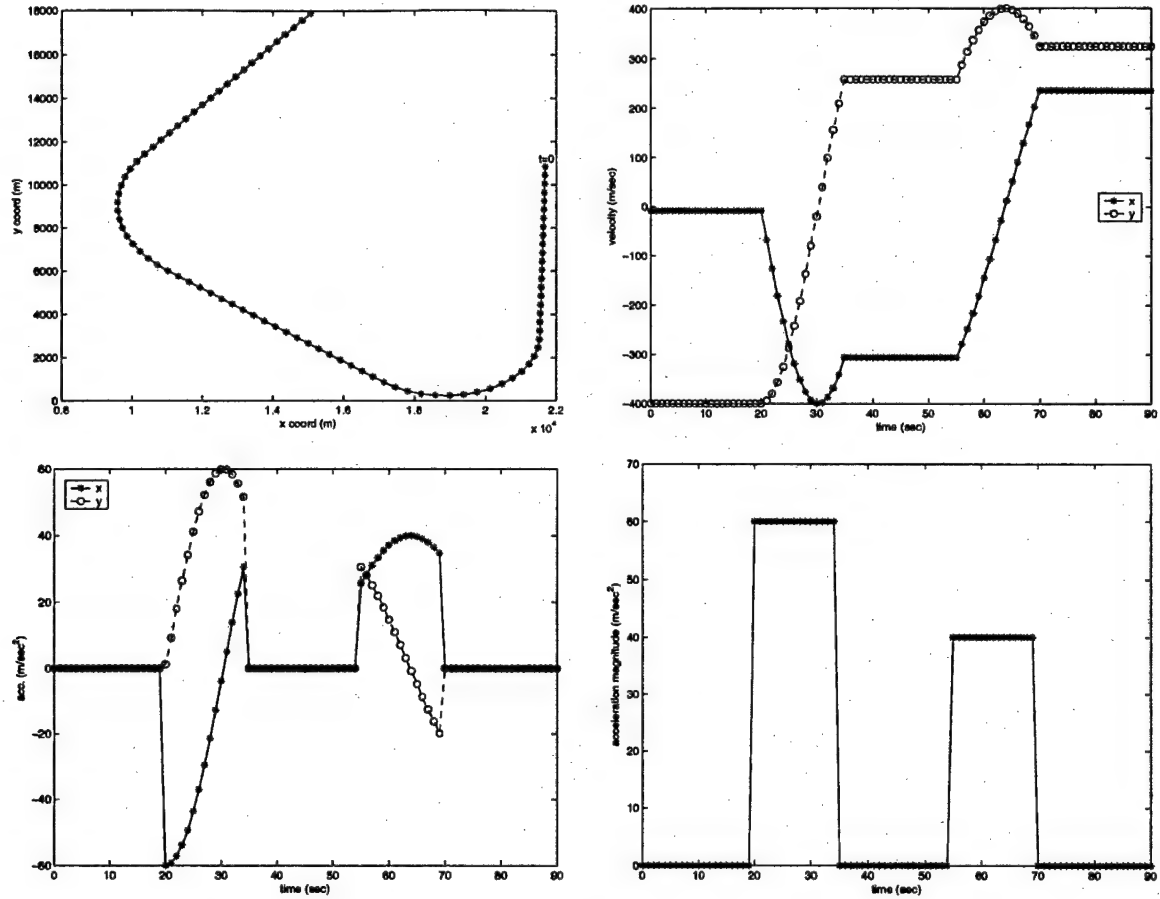


Figure 1: Trajectory of maneuvering target (read left to right, top to bottom). (a) Position in  $xy$  plane. (b)  $x$  and  $y$  velocities. (c)  $x$  and  $y$  accelerations. (d) magnitude of accelerations

always applies. It is seen from Fig. 4 that when the unknown but fixed timing mismatch  $d_k$  is more than one fifth of the sampling time, the performance improvement is significant compared with the standard IMMSPDAF algorithm that ignores the time-delay  $d$ .

## VI Conclusions

We investigated an IMMSPDAF algorithm with asynchronous measurement (there is unknown but fixed timing mismatch between sensor platforms) for tracking a highly maneuvering target in clutter. The proposed algorithm was illustrated via a simulation example where it outperformed a standard IMMSPDAF algorithm that ignored the possible timing mismatch (especially when the possible timing mismatch is more than one fifth of the sampling time).

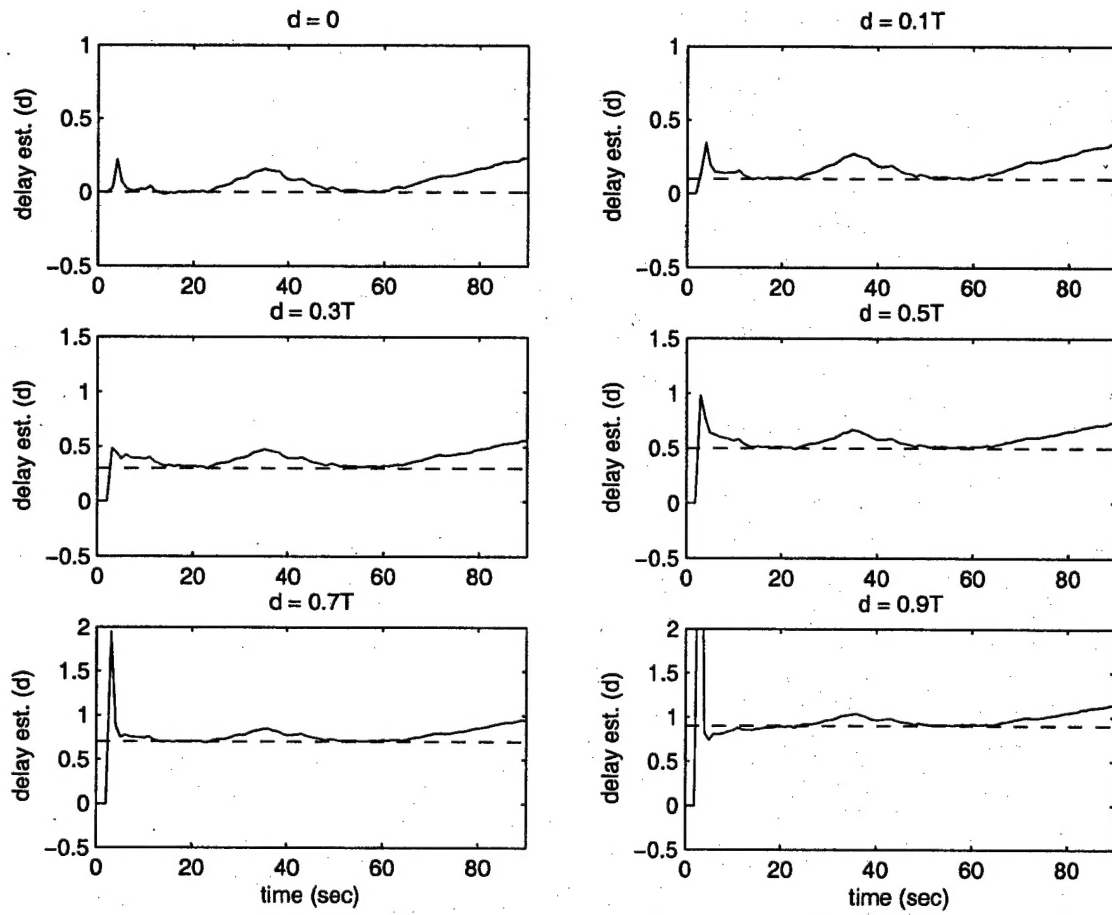


Figure 2: Estimation of delay (given unknown but fixed timing mismatch between two separated sensors) based on 100 Monte Carlo runs (read left to right, top to bottom). (a)  $d = 0$ . (b)  $d = 0.1T$ . (c)  $d = 0.3T$ . (d)  $d = 0.5T$ . (e)  $d = 0.7T$ . (f)  $d = 0.9T$ . ( $T$  = sampling rate)



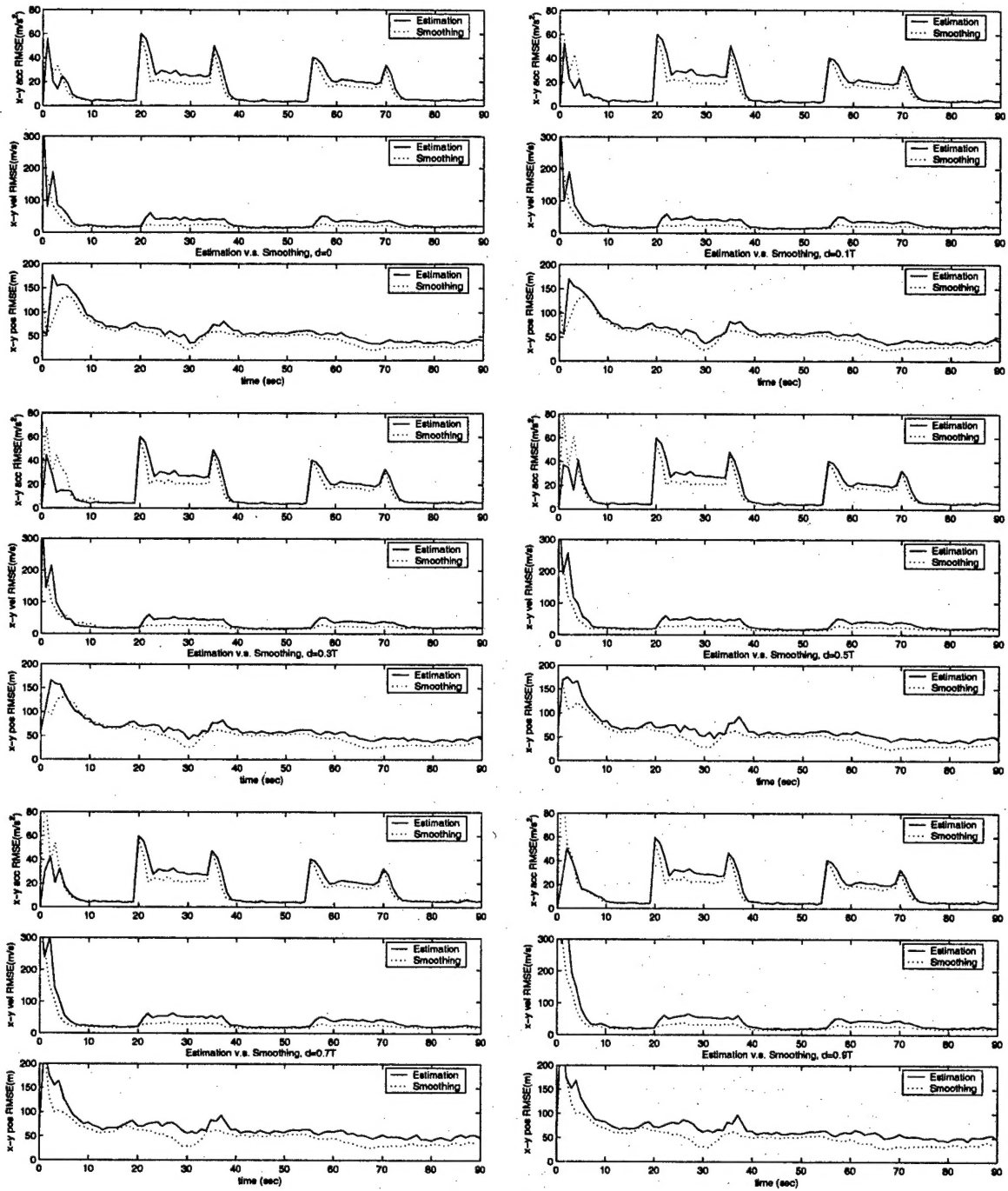


Figure 3: Comparison of filtering and smoothing (lag=1) for various delay values (acceleration, velocity, and position RMS errors (3 rows each), read left to right, top to bottom). (a)  $d = 0$ . (b)  $d = 0.1T$ . (c)  $d = 0.3T$ . (d)  $d = 0.5T$ . (e)  $d = 0.7T$ . (f)  $d = 0.9T$ . ( $T$  = sampling rate). In the figure legends, estimation refers to filtering and smoothing is with lag=1.

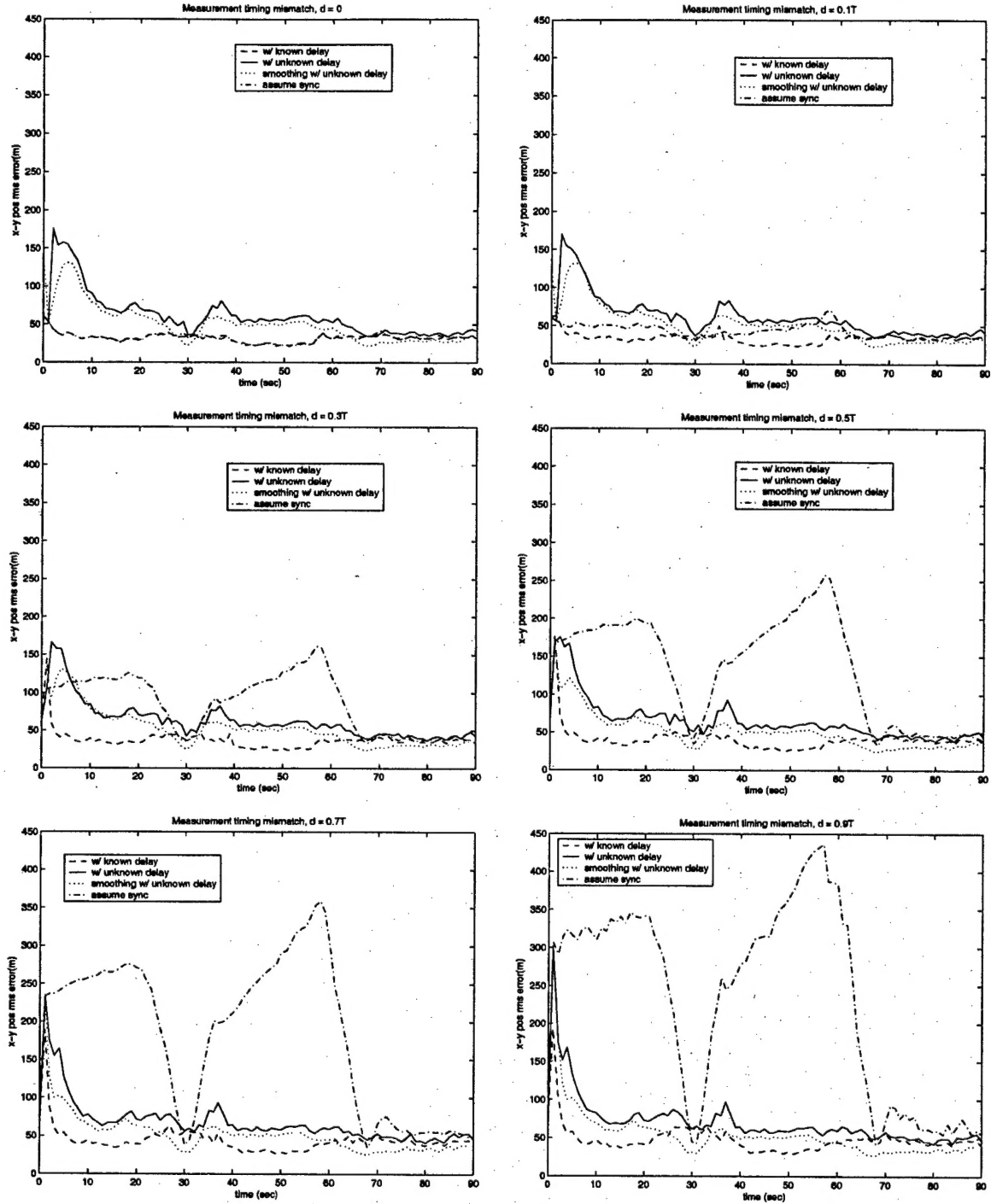


Figure 4: RMSE in position using IMMSPDAF under various scenarios of known delay, estimated delay and ignoring delay, for various delay values (read left to right, top to bottom). (a)  $d = 0$ . (b)  $d = 0.1T$ . (c)  $d = 0.3T$ . (d)  $d = 0.5T$ . (e)  $d = 0.7T$ . (f)  $d = 0.9T$ . ( $T$  = sampling rate). Unless otherwise stated, the results are for filtering.

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